Solutions to JEE Advanced Home Practice Test -7 | JEE 2024 | Paper-2

PHYSICS

1.(BC) Each object has a speed of $v_0 = \sqrt{2gh}$ when arranging at the rigid, horizontal ground. The lower ball of mass m_1, arriving first, rebounds with an upward velocity of the same magnitude v_0 since the collision is elastic. It then collides with the ball of mass m_2 still travelling downwards at a speed of v_0.

Hence, the velocities of the two objects after the collision are (with upward speeds taken as positive)

$$u_1 = 2\cdot\frac{m_1v_0 - m_2v_0}{m_1+m_2} - v_0 = v_0\cdot\frac{m_1-3m_2}{m_1+m_2} \qquad \ldots\text{(i)}$$

$$u_2 = 2\cdot\frac{m_1v_0 - m_2v_0}{m_1+m_2} + v_0 = v_0\cdot\frac{3m_1-m_2}{m_1+m_2} \qquad \ldots\text{(ii)}$$

(a) To make the mass m_1 stay at rest after the second collision (the collision with the upper object), we need $u_1 = 0$. Therefore it follows from (i) that

$$m_1 - 3m_2 = 0$$

That is, the ratio of the masses in question is:

$$\frac{m_1}{m_2} = 3.$$

(b) Then the speed of the rebounding ball obtained from (2) is:

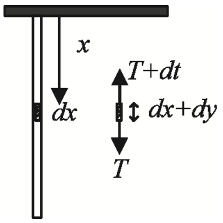
$$u_2 = v_0\frac{9m_2 - m_2}{3m_2 + m_2} = 2v_0$$

and the maximum height reached can be calculated from the law $h = \frac{v^2}{2g}$:

$$h_1 = \frac{4v_0^2}{2g} = 4\cdot\frac{v_0^2}{2g} = 4h$$

2.(AC)

Selecting an element of length 'dx' at depth x, as shown. The tension force under which it is going to elongate is the weight of rod hanging under it



$$T(x) = \lambda(l-x)g$$

$$stress = \sigma(x) = \frac{\lambda(l-x)g}{A}$$

$$\because \gamma = \frac{\sigma(x)}{\in(x)} \Rightarrow \text{ strain at } x = \in(x) = \frac{\lambda(l-x)g}{\gamma A}$$

Let the element stretches by 'dy' then $\in = \frac{dy}{dx}$; $\quad \frac{dy}{dx} = \frac{\lambda g}{\gamma A}(l-x)$

Total elongation till depth $x = y_x = \int_0^x \frac{\lambda g}{\gamma A}(l-x)\,dx = \frac{\lambda g}{\gamma A}\left(lx - \frac{x^2}{2}\right)$

So, total elongation of whole rod $= y_l = \frac{\lambda g}{\gamma A}\left(l^2 - \frac{l^2}{2}\right) = \frac{\lambda g l^2}{2\gamma A}$

Work done by gravity on selected element = $dm.g.y_x = \lambda dx.g.\frac{\lambda g}{\gamma A}\left(lx - \frac{x^2}{2}\right)$

Total work one by gravity on rod till depth $x = \frac{\lambda^2 g^2}{\gamma A}\int_0^x\left(lx - \frac{x^2}{2}\right)dx$

$$W_{g(x)} = \frac{\lambda^2 g^2}{\gamma A}\left[\frac{lx^2}{2} - \frac{x^3}{6}\right]$$

So, total work done by gravity on whole rod = $W_{g(l)} = \frac{\lambda^2 g^2}{\gamma A}\left[\frac{l^3}{2} - \frac{l^3}{6}\right] = \frac{\lambda^2 g^2 l^3}{3\gamma A}$

Work done by gravity on upper half $= W_g\left(l/2\right) = \frac{\lambda^2 g^2}{\gamma A}\left[\frac{l^3}{8} - \frac{l^3}{48}\right] = \frac{\lambda^2 g^2}{\gamma A}.\frac{5l^3}{48} = \frac{5\lambda^2 g^2 l^3}{48\gamma A}$

Work done by gravity on lower half $= W_g\left(\frac{l}{2} \to l\right) = \frac{\lambda^2 g^2 l^2}{3\gamma A} - \frac{5\lambda^2 g^2 l^3}{48\gamma A} = \frac{11}{48}\frac{\lambda^2 g^2 l^3}{\gamma A}$

Elastic potential energy stored in selected element $dU_e = \frac{1}{2}\sigma\varepsilon dv = \frac{\sigma^2 .Adx}{2\gamma}$

$$dU_e = \frac{\lambda^2 (l-x)^2 g^2}{A^2.2\gamma}A.dx = \frac{\lambda^2 g^2 (l-x)^2 dx}{2A\gamma}$$

Elastic potential energy stored till depth $x = U_e(x) = \frac{\lambda^2 g^2}{2A\gamma}\int_{x=0}^{x}(l-x)^2 dx = \frac{\lambda^2 g^2}{2A\gamma}\int_{x=0}^{x}\left(l^2 + x^2 - 2lx\right)dx$

$$= \frac{\lambda^2 g^2}{2A\gamma}\left[l^2 x + \frac{x^3}{3} - lx^2\right]$$

Elastic potential energy stored in whole rod $= U_e(l) = \frac{\lambda^2 g^2}{2A\gamma}\left[l^3 + \frac{l^3}{2} - l^3\right] = \frac{\lambda^2 g^2 l^3}{6A\gamma}$

Elastic potential energy stored in upper half $= U_e(l/2) = \frac{\lambda^2 g^2}{2A\gamma}\left[\frac{l^3}{2} + \frac{l^3}{24} - \frac{l^3}{4}\right] = \frac{7\lambda^2 g^2 l^3}{48A\gamma}$

Elastic potential energy stored in lower half $= \frac{\lambda^2 g^2 l^3}{A\gamma}\left[\frac{1}{6} - \frac{7}{48}\right] = \frac{1\lambda^2 g^2 l^3}{48A\gamma}$

Heat liberated = work done by gravity – elastic energy stored $= \dfrac{\lambda^2 g^2 l^3}{3A\gamma} - \dfrac{\lambda^2 g^2 l^3}{6A\gamma} = \dfrac{\lambda^2 g^2 l^3}{6A\gamma} = Q$

$$Q = ms.\Delta T \Rightarrow \frac{\lambda^2 g^2 l^3}{6A\gamma} = \lambda \ell.S_0.\Delta T \Rightarrow \Delta T = \frac{\lambda g^2 l^2}{6A\gamma S_0}$$

$$\Delta T \propto l^2; \qquad \frac{\Delta T_1}{\Delta T_2} = \frac{l_0^2}{(2l_0)^2} = \frac{1}{4}$$

3.(BC) **(B)** $\Delta\Phi = B(r,t).2r\pi.\Delta r.$

According to the condition given in the problem $\Delta\Phi = \dfrac{E_0}{r} t.2\pi r\Delta r.$

Let us sum up the elementary fluxes:

$$\Phi(t) = \sum \Delta\Phi = 2\pi E_0 t \sum_i \Delta r_i = 2\pi E_0 t R,$$

$$\Phi(t) = 2\pi E_0 R t \qquad (1)$$

$$E(R) = \frac{1}{2\pi R}.\frac{\Delta\Phi}{\Delta t} = \frac{1}{2\pi R}.\frac{2\pi R E_0 \Delta t}{\Delta t} = E_0 \qquad (2)$$

The bead experiences a constant electric field in the direction of the tangent, so according to (2), its velocity as function of time is

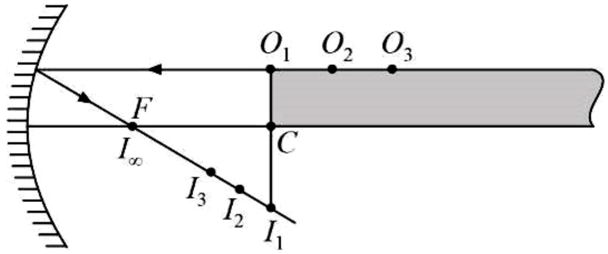
$$v(t) = \frac{E_0 q}{m}.t. \qquad (3)$$

(C) Applying the fundamental law of dynamics in radial direction: $qvB + N = m\dfrac{v^2}{R}$

Where N is the normal force exerted by the track. Using (3), its magnitude is

$$N = m\frac{v^2}{R} - qvB = \frac{m}{R}.\frac{E_0^2 q^2 t^2}{m^2} - q.\frac{E_0 q}{m}t.\frac{E_0 t}{R} = 0.$$

4.(BC) If we draw an incident ray along the top side of rectangular strip, which happens to be parallel to the principal axis. After reflection this ray passes through focus. Thus, image of all points on the top surface of the strip $O_1, O_2, O_3, \ldots\ldots$ etc lie on this reflected ray at locations $I_1, I_2, I_3, \ldots\ldots$ etc in between focus and centre of curvature. Hence image formed is triangle in shape.



5.(AD) Let a and b, the number of α and β^- particles are emitted when ${}^{238}_{92}U$ decay to ${}^{206}_{82}Pb$.

We know that

(i) The emission of a α – particle $({}^4_2He)$ decreases the charge number by two and mass number by four Thus emission of a, α-particles reduce the charge number by $2a$ and mass number by $4a$.

(ii) The emission of β-particle increase the charge number by one and leaving the mass number unchanged.

Thus emission of b, β-particles increases the charge number by $b \times 1 = b$.

Thus, $^{238}_{92}U \rightarrow {}^{206}_{82}Pb + a({}^{4}_{2}He) + b({}^{0}_{-1}\beta)$

Applying the law of conservation of charge number and mass number before and after the decay, we have

$$92 = 82 + 2a - b$$

and $238 = 206 + 4a$

Solving, $a = 8$

$b = 6$

6.(AC) $f_{\text{recieved by wall}} = f_{\text{source}}\left(\dfrac{330+3.3}{330}\right) = 1010\ Hz$

Now the wall reflects this sound. It acts as a source moving at $3.3\ m/s^{-1}$ and emits sound at a frequency of 1010 Hz.

$f_{\text{heard by driver}} = f_{\text{wall as source}}\left(\dfrac{330}{330-3.3}\right) = 1020\ Hz$

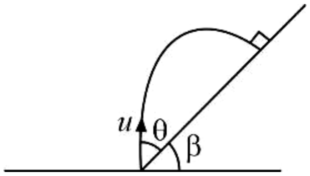
7.(10) Viscous force [$v =$ instantaneous speed of the car]

$F_v = \eta A \dfrac{dv}{dh} = \eta A \dfrac{v}{h}$ $\quad\therefore\quad$ $M\dfrac{dv}{dt} - \eta A \dfrac{v}{h}$ $\quad$ or, $Mv\dfrac{dv}{dx} = -\eta A \dfrac{v}{h}$

Or, $\displaystyle\int_{v_0}^{0} dv = -\frac{\eta A}{hM}\int_0^x dx$ $\quad\Rightarrow v_0 = \dfrac{\eta A}{hM}x$ $\quad\Rightarrow\quad x = \dfrac{hMv_0}{\eta A}$

$$x = \frac{10^{-4} \times 10^3 \times 20}{10^{-3} \times 0.2} = 10^4\, m = 10\, km\, (!)$$

8.(3.5) For ball to retrace path, it must hit the inclined plane at 90°



$\therefore$ $v_x = 0 \rightarrow 2\tan\theta\tan\beta = 1$

$R = u_x T - \dfrac{1}{2} g \sin\beta T^2$

On solving $u = \sqrt{\dfrac{gR(1+3\sin^2\beta)}{2\sin\beta}}$

Put in the values, $u = 3.5\ m/s$

9.(4) By an external force in case of SHM only equilibrium position changes. Time period remains same. As speed of block at mean position is same in each case therefore amplitude will be same in all cases. In case-4 equilibrium position $x_0 = 3\,mg/k$ which is maximum among all cases.

$v_{\max} = A\omega$

$A = \dfrac{v_{\max}}{\omega}$

Since ω is same in all cases, A is also same.

Max. elongation = (elongation at mean position) + A

10.(80) $i = \dfrac{\theta}{\text{sensitivity}}$

$$i_{\text{full deflection}} = \frac{0.1}{10} = 10mA$$

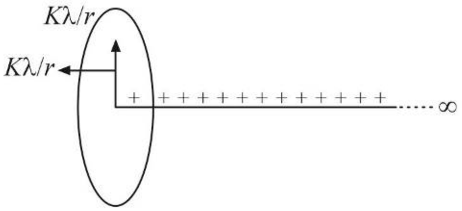
For conversion to voltmeter

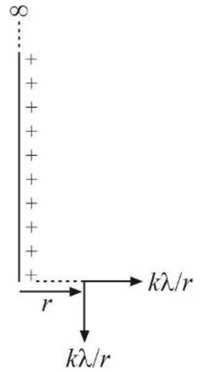
$V = i_{\text{full deflection}}(R_G + R)$

$1 = 0.01(20 + R)$

$R = 80\Omega$ should be connected in series

11.(0.50)





Electric field due to wire has 2 components, only one contributes to flux, the other is perpendicular to area vector.

$$\phi = \int \vec{E} \cdot d\vec{A} = \int_0^R \frac{K\lambda}{r} 2\pi r dr = \frac{\lambda}{2 \in_0} \int_0^R dr \ ; \qquad \phi = \frac{\lambda R}{2 \in_0}$$

12.(–0.97)

We use $P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$$\Rightarrow \qquad P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma} = 10^5 \left(\frac{6}{2}\right)^{1.67} \qquad \Rightarrow \qquad P_2 = 10^5 (3)^{1.67}$$

Work done by gas in adiabatic process is

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{10^5(6 - 2(3)^{1.67})}{0.67} \qquad = -973.13\ kJ$$

13.(12.43)

The maximum kinetic energy of the electrons immediately upon ejection is the difference between the energy of the incident photon and the threshold energy.

$$K = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

This kinetic energy of ejected electron is converted to electrostatic potential energy, $\Delta U = eEd$, as electrons come to rest while moving in the direction of electric field. Therefore, $K = Eed$.

and $$\lambda_0 = \left(\frac{1}{\lambda} - \frac{eEd}{hc}\right)^{-1} \qquad \rightarrow \lambda_0 = 2\lambda \rightarrow d = \frac{hc}{2\lambda eE} = 12.43\ m$$

14.(0.25) Intensity of light on the mirror surface $l = \frac{W}{4\pi R^2}$

Consider a small patch of area dS on the surface of the mirror. Energy incident per unit time on this area is

$$dE = lds = \frac{WdS}{4\pi R^2}$$

Momentum incident on area $dS = \frac{dE}{c} = \frac{WdS}{4\pi R^2 c}$

Light is reflected back towards the centre of the sphere, hence change in momentum per unit time for area dS is

$$= \frac{2WdS}{4\pi R^2 c}$$

This is equal to force on dS. $\quad dF = \frac{2WdS}{4\pi R^2 c}$

By symmetry the resultant force is along X direction $\quad \therefore \quad dF_x = dF\cos\theta = \frac{2W(dS\cos\theta)}{4\pi R^2 c}$

Projection of dS on vertical plane is $dA = dS\cos\theta$

$$\therefore \quad F_x = \frac{2W}{4\pi R^2 c}\int dS\cos\theta = \frac{2W}{4\pi R^2 c}\int dA \quad = \frac{2W}{4\pi R^2 c}\pi\left(\frac{d}{2}\right)^2 = \frac{Wd^2}{8R^2 c}$$

15.(B) Standard current carrying distributions can be used to calculate dependence.

16.(D) Let a be the semi-major axis, $r_{\text{perihelion}} = a(1-e)$

$$v_{\text{perihelion}} = \sqrt{\frac{Gm}{a}\left(\frac{1+e}{1-e}\right)}; \qquad v_1 = \sqrt{\frac{Gm_1}{a}\times\frac{1.6}{0.4}} = \sqrt{\frac{4Gm_1}{a}}$$

$$v_2 = \sqrt{\frac{Gm_2}{a}\times\frac{1.8}{0.2}} = \sqrt{\frac{9Gm_2}{a}}; \qquad \frac{v_1}{v_2} = \frac{2}{3}\sqrt{\frac{m_1}{m_2}} = \frac{2}{3}\sqrt{4} = \frac{4}{3}$$

$$\frac{k_1}{k_2} = \frac{m_1 v_1^2}{m_2 v_2^2} = 4\times\frac{4}{9}\times 2 = \frac{32}{9}$$

$$\frac{T_1}{T_2} = \frac{m_2}{m_1} = 1, \text{ (Since a is same for both)}$$

$$\frac{L_1}{L_2} = \left(\frac{m_1 v_1 r_1}{m_2 v_2 r_2}\right) \text{ at perihelion } = 4\times\frac{4}{3}\times\frac{(1-0.6)}{(1-0.8)} = \frac{32}{3}$$

17.(A) Work done is positive in clockwise process and negative in anti-clockwise process.

18.(B) (P) $\quad \vec{v} = \alpha\hat{i} + \beta\hat{j}$

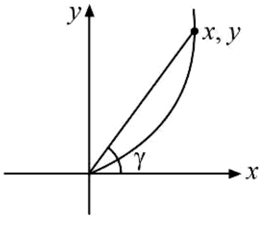
Speed is constant. Particle moves in straight line passing through origin, $\omega = 0$ always.

(Q) $\quad$ uniform circular motion both v and ω are constant.

(S) $\quad x = \alpha t,\ y = \frac{\beta t^2}{2}$

$$y = \frac{\beta}{2\alpha^2}x^2 \rightarrow \text{parabolic path}$$

$$\vec{v} = \alpha\hat{i} + \beta t\hat{j} \rightarrow \text{ speed is increasing}$$



$$\omega = \frac{d\gamma}{dt}$$

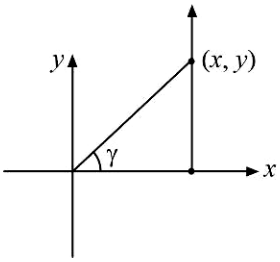
$$\tan\gamma = \frac{y}{x} = \frac{\beta t}{2\alpha}$$

$$\sec^2\gamma \frac{d\gamma}{dt} = \frac{\beta}{2\alpha}$$

$$\omega = \frac{\beta}{2\alpha}\cos^2\gamma = \frac{\beta\alpha}{2(\beta^2 t^2 + 4\alpha^2)}$$

ω decreases with time.

(R) $\vec{v} = \beta\hat{j}$



$$\tan\gamma = \frac{y}{x} = \frac{\beta t}{\alpha}$$

$$\sec^2\gamma \frac{d\gamma}{dt} = \frac{\beta}{\alpha}$$

$$\omega = \frac{\beta}{\alpha}\cos^2\gamma$$

$$= \frac{\beta}{(\alpha^2 + \beta^2 t^2)}$$

ω decreases with time

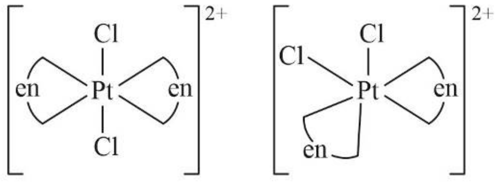
CHEMISTRY

1.(ACD)

Let oxidation number of Pt = x

$(1 \times x + 2 \times 0) + 2 \times (-1) = +2$

$\Rightarrow \quad x = +4$



Trans form (Optically inactive) cis form (Optically active)

2.(BC) $Ag^+ + HCl \longrightarrow AgCl\downarrow + H^+$

The precipitate dissolve in concentrated HCl, $AgCl\downarrow + Cl^- \longrightarrow AgCl_2^-$

When KCN is added in small amount the Cu^{2+} ions first form a yellow precipitate of $Cu(CN)_2$ which on standing decomposes to white precipitate of CuCN.

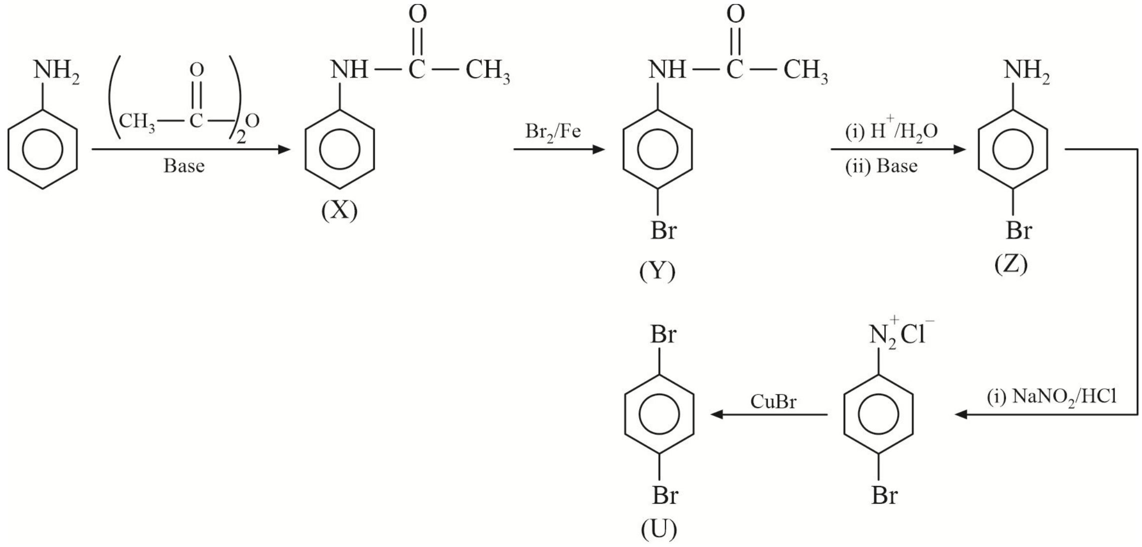
$$Cu^{2+} + 2CN^- \longrightarrow \underset{\text{(yellow ppt.)}}{Cu(CN)_2\downarrow}$$

$$\underset{\text{yellow}}{2Cu(CN)_2} \longrightarrow \underset{\text{white}}{2CuCN\downarrow} + (CN)_2$$

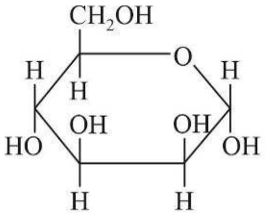
$$Hg^{2+} + \underset{(CH_3COO)_2Co}{\underset{\uparrow}{Co^{2+}}} + \underset{NH_4SCN}{\underset{\uparrow}{4SCN^-}} \longrightarrow \underset{\text{Blue precipitate}}{Co[Hg(SCN)_4]}$$

$$\underset{\text{Black precipitate}}{BiI_3\downarrow} + H_2O \longrightarrow \underset{\text{Orange}}{BiOI\downarrow} + 2H^+ + 2I^-$$

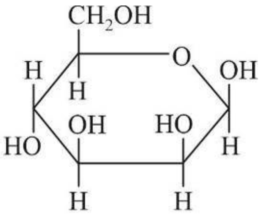
3.(ABC)



4.(BD) The structure of D-Mannose is



(α-form)



(β-form)

5.(CD) For first order kinetics, rate = kC

$$C = C_0 e^{-kt}; \qquad k = \frac{2.303}{t}\log\frac{C_0}{C}$$

At $\quad t = t_{3/4},\ C = \frac{C_0}{4}; \qquad t_{3/4} = \frac{2.303}{k} \times \log\frac{C_0}{\left(\frac{C_0}{4}\right)}$

$$t_{3/4} = \frac{2.303}{k} \times \log 4; \qquad t_{3/4} = \frac{1.386}{k}$$

$$-\log C + \log C_0 = \frac{k}{2.303} \cdot t; \qquad \log C = -\frac{k}{2.303} \cdot t + \log C_0$$

6.(ABCD)

$$T_i = \frac{P_i V_i}{nR}; \qquad T_i = \frac{1 \times 1}{1 \times R} = \frac{1}{R}$$

$$T_f = \frac{P_f V_f}{nR}; \qquad T_f = \frac{1 \times 2}{1 \times R} = \frac{2}{R}; \ T_f = 2T_i$$

$$\Delta U = nC_V\ \Delta T = 1 \times \frac{3}{2} \times R \times (T_f - T_i)$$

$$\Delta U = \frac{3}{2} \times R \times T_i = \frac{3}{2} \times P_i V_i$$

$$\Delta U = \frac{3}{2} \times 1 \times 1 = \frac{3}{2} \text{ litre bar} = \frac{3}{2} \times 100 \text{ Joule}$$

$$\Delta U = 150 \text{ Joule}$$

H = U + PV

$$\Delta H = \Delta U + \Delta(PV) = \frac{3}{2} P_i V_i + (P_f V_f - P_i V_i)$$

$$\Delta H = \left(\frac{3}{2} + 1\right) \text{ litre bar}$$

$$\Delta H = \frac{5}{2} \times 100 \text{ J}; \qquad \Delta H = 250 \text{ J}$$

Entropy change is positive
Work done by gas is zero

7.(4) $SO_2,\ SO_3,\ H_3PO_4$ and $HClO_4$
has $p\pi - d\pi$ bonding.

8.(130) Moles of $Na[Ag(CN)_2] = \frac{500}{1000} \times 0.5 = 0.25 = \frac{1}{4}$

$$Zn + 2Na\,\underset{\frac{1}{4}}{[Ag(CN)_2]} \longrightarrow Na_2[Zn(CN)_4] + 2Ag$$

Moles of Zn required $= \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Mass of Zn required = x $= \frac{1}{8} \times 65 = \frac{65}{8}$ gram

Hence $16x = \frac{65}{8} \times 16 = 130$

9.(110) $CaOCl_2 + H_2O \longrightarrow Ca(OH)_2 + Cl_2$

$Cl_2 + 2KI \longrightarrow 2KCl + I_2$

$I_2 + 2Na_2S_2O_3 \longrightarrow Na_2S_4O_6 + 2NaI$

m.eq of $Na_2S_2O_3 = 0.125 \times 20 = 2.5$

m.eq of I_2 = m.eq of $Na_2S_2O_3 = 2.5$

m.eq of Cl_2 = m.eq of $I_2 = 2.5$

m.eq of Cl_2 in 100 ml solution $= 2.5 \times \frac{100}{25} = 10$

Mass of chlorine $= \frac{10}{1000} \times \frac{71}{2}$ gram $= 0.355$ gram

% of chlorine = x $= \frac{0.355}{3.55} \times 100 = 10$

Hence 11x = 110

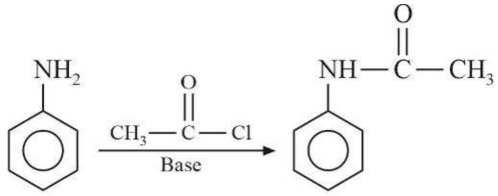
10.(256)

Number of stereo centres n = 4

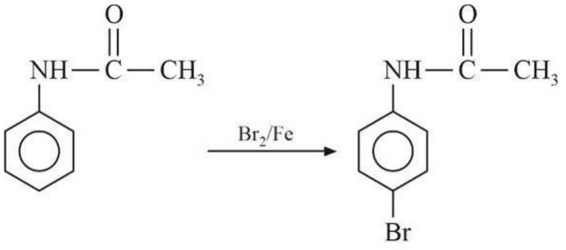
Hence, number of stereo isomers $x = 2^n = 2^4 = 16$

Therefore, 16x = 16 × 16 = 256

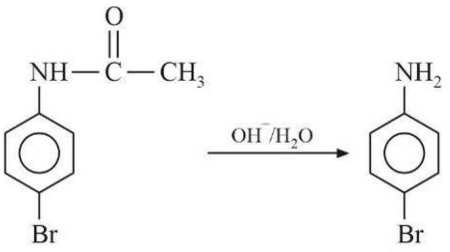
11.(236)



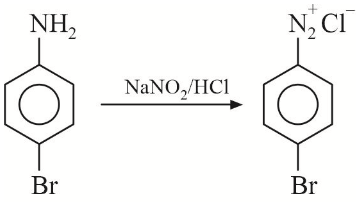
(A)



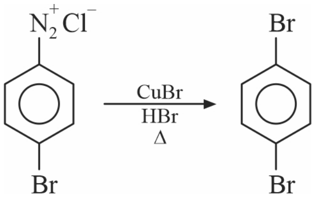
(B)



(C)



(D)



Moles of aniline = 25

Moles of A = 25 × 0.4 = 10

Moles of B = 10 × 0.5 = 5

Moles of C = 5 × 0.4 = 2

Moles of D = 2 × 0.5 = 1

Moles of E = 1

Mass of E = 1 × 236 = 236 gram

12.(50) $NO_{(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons NO_{2(g)}$

$$\Delta G^\circ = \Delta G^\circ_f(NO_2)_{(g)} - \Delta G^\circ_f(NO)_{(g)}$$

$$= 52 - 87$$

$$\Delta G^\circ = -35 \text{ kJ}$$

$$\Delta G^\circ = -2.303 \, RT \, \log K_P$$

$$-35 \times 1000 = -2.303 \times 8.314 \times 298 \, \log K_P$$

$$\Rightarrow \quad \log K_P = 6.13 \quad \Rightarrow \quad K_P = 1.34 \times 10^6$$

	$NO_{(g)}$	+ $\frac{1}{2}O_{2(g)}$	$\rightleftharpoons$ $NO_{2(g)}$
Initially	1	11/4	0
Equilibrium	p	9/4	1

$$K_p = 1.34 \times 10^6 = \frac{p_{NO_2}}{p_{NO} \times p_{O_2}^{1/2}} \quad \Rightarrow \quad 1.34 \times 10^6 = \frac{1}{p \times \left(\frac{9}{4}\right)^{1/2}}$$

$\Rightarrow \quad p = 5 \times 10^{-7} \quad \Rightarrow \quad p = 50 \times 10^{-8}$; Hence, $x = 50$

13.(1727.25)

$$A \underset{K_b}{\overset{K_f}{\rightleftharpoons}} B$$

$$\begin{array}{lll} t=0 & [A]_0 & 0 \\ t=t & [A]_0 - x & x \\ t=t_{eq} & [A]_0 - x_{eq} & x_{eq} \end{array}$$

$[A]_{eq} = [A]_0 - x_{eq}$

$[B]_{eq} = x_{eq}$

$$(k_f + k_b) = \frac{1}{t}\ln\left(\frac{x_{eq}}{x_{eq} - x}\right)$$

$$(k_f + k_b) = \frac{6}{\ln 2}\ln\frac{[B]_{eq}}{[B]_{eq} - \frac{[B]_{eq}}{2}}$$

$K_f + K_b = 6 \qquad \ldots\ldots(i)$

$K_b - K_f = 2 \qquad \ldots\ldots(ii)$

On solving (i) and (ii)

$K_b = 4,\ K_f = 2$

$K_{eq} = \frac{K_f}{K_b} = \frac{1}{2}$; $\Delta G° = -RT \ln K_{eq}$

$\Delta G° = -2.303\ RT \log K_{eq}$

$\Delta G° = -2.303 \times 2500 \log\frac{1}{2}$ Joule

$\Delta G° = 1727.25$ Joule

14.(30.88)

$E^{\circ}_{25°C} = 0.3525$ V; $\qquad E^{\circ}_{20°C} = 0.3533$ V

$$\frac{dE}{dT} = \frac{E^{\circ}_{25°C} - E^{\circ}_{20°C}}{(T_2 - T_1)} = \frac{0.3525 - 0.3533}{(25 - 20)}$$

$$\frac{dE}{dT} = -1.6 \times 10^{-4}\ \text{volt/K}$$

$$\Delta S° = nF\frac{dE}{T} = 2 \times 96500 \times (-1.6 \times 10^{-4})\ \text{J/K}$$

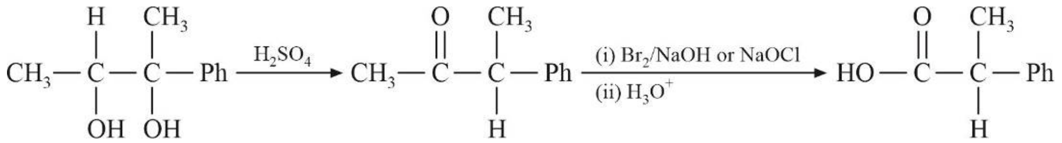
$\Delta S° = -30.88$ J/K

15.(B) $d^2sp^3 \rightarrow [Fe(en)_3]^{3+}, [Co(C_2O_4)_3]^{3-}$

$sp^3 \rightarrow Na_2[Zn(CN)_4], [Ni(CO)_4]$

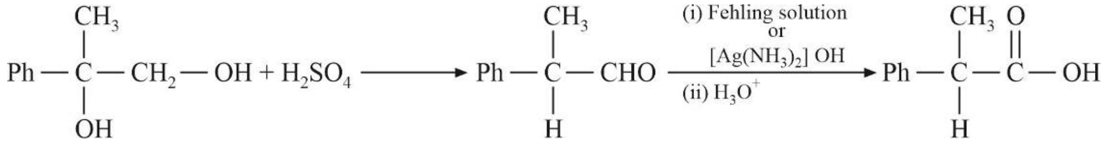
$sp^3d^2 \rightarrow [NiCl_6]^{2-}$; $\qquad dsp^2 \rightarrow K_2[PtCl_4]$

16.(C) (P)



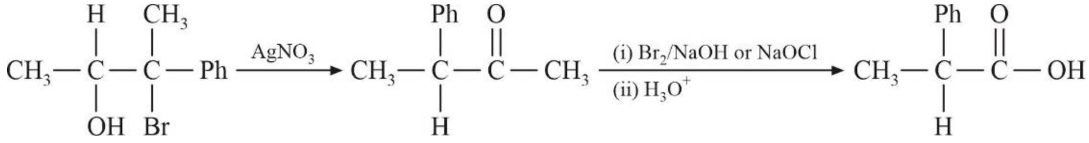
$$CH_3-\underset{OH}{\overset{H}{C}}-\underset{OH}{\overset{CH_3}{C}}-Ph \xrightarrow{H_2SO_4} CH_3-\overset{O}{\overset{\|}{C}}-\underset{H}{\overset{CH_3}{C}}-Ph \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } Br_2/NaOH \text{ or } NaOCl} HO-\overset{O}{\overset{\|}{C}}-\underset{H}{\overset{CH_3}{C}}-Ph$$

(Q)



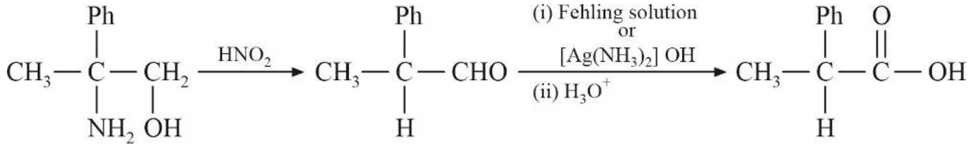
$$Ph-\underset{OH}{\overset{CH_3}{C}}-CH_2-OH+H_2SO_4 \longrightarrow Ph-\underset{H}{\overset{CH_3}{C}}-CHO \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) Fehling solution or } [Ag(NH_3)_2]\,OH} Ph-\underset{H}{\overset{CH_3}{C}}-\overset{O}{\overset{\|}{C}}-OH$$

(R)



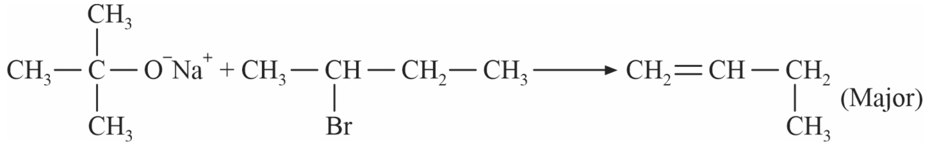
$$CH_3-\underset{OH}{\overset{H}{C}}-\underset{Br}{\overset{CH_3}{C}}-Ph \xrightarrow{AgNO_3} CH_3-\underset{H}{\overset{Ph}{C}}-\overset{O}{\overset{\|}{C}}-CH_3 \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } Br_2/NaOH \text{ or } NaOCl} CH_3-\underset{H}{\overset{Ph}{C}}-\overset{O}{\overset{\|}{C}}-OH$$

(S)



$$CH_3-\underset{NH_2}{\overset{Ph}{C}}-\underset{OH}{CH_2} \xrightarrow{HNO_2} CH_3-\underset{H}{\overset{Ph}{C}}-CHO \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) Fehling solution or } [Ag(NH_3)_2]\,OH} CH_3-\underset{H}{\overset{Ph}{C}}-\overset{O}{\overset{\|}{C}}-OH$$

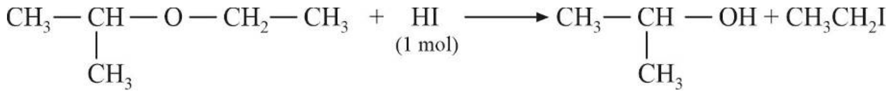
17.(A) (P)



$$CH_3-\underset{CH_3}{\overset{CH_3}{C}}-\overset{-}{O}\overset{+}{Na}+CH_3-\underset{Br}{CH}-CH_2-CH_3 \longrightarrow CH_2{=}CH-\underset{CH_3}{CH_2} \text{ (Major)}$$

(Q) $CH_3CH_2CH_2CH_2OH \xrightarrow{H_2SO_4/\Delta} CH_3-CH=\underset{\text{(Major)}}{CH}-CH_3$

(R)



$$CH_3-\underset{CH_3}{CH}-O-CH_2-CH_3+\underset{\text{(1 mol)}}{HI} \longrightarrow CH_3-\underset{CH_3}{CH}-OH+CH_3CH_2I$$

(S)

$$CH_3-\underset{CH_3}{CH}-OH+HI \longrightarrow CH_3-\underset{CH_3}{CH}-I+H_2O$$

18.(C) (P)

$$NH_4OH + HCl \longrightarrow NH_4Cl + H_2O$$

	NH_4OH	HCl	NH_4Cl	H_2O
Initially	20	10	–	
Finally	10	0	10	

$pOH = pK_b$

$pH = 14 - pK_b$

Hence $[H^+]$ concentration does not change on dilution.

(Q)

$$CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$$

	CH_3COOH	$NaOH$	CH_3COONa	H_2O
Initially	20	20	–	
Finally	0	0	20	

$$[OH^-]=\sqrt{\frac{C\,K_w}{K_a}}\,; \qquad [H^+]=\frac{K_w}{[OH^-]}=\sqrt{\frac{K_wK_a}{C}}$$

(R)

$$\begin{array}{lccccccc}
 & CH_3COOH & + & NaOH & \longrightarrow & CH_3COONa & + & H_2O \\
\text{Initially} & 20 & & 40 & & - & & \\
\text{Finally} & 0 & & 20 & & 20 & &
\end{array}$$

Hence $[OH^-]$ will be controlled by NaOH

(S)

$$\begin{array}{lccccccc}
 & NH_4OH & + & HCl & \longrightarrow & NH_4Cl & + & H_2O \\
\text{Initially} & 10 & & 40 & & - & & \\
\text{Finally} & 0 & & 30 & & 10 & &
\end{array}$$

Hence $[H^+]$ is controlled by HCl

MATHEMATICS

1.(ABC) $f(n)=\sum_{r=1}^{n}\tan^{-1}\left(\frac{2r}{r^4+r^2+2}\right)$

$$=\sum_{r=1}^{n}\tan^{-1}\left(\frac{(r^2+r+1)-(r^2-r+1)}{1+(r^2+r+1)\,(r^2-r+1)}\right)$$

$$=\sum_{r=1}^{n}\left[\tan^{-1}(r^2+r+1)-\tan^{-1}(r^2-r+1)\right]$$

$$f(n)=\tan^{-1}(n^2+n+1)-\frac{\pi}{4}$$

$$\lim_{n\to\infty}f(x)=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4};\quad \lim_{n\to\infty}f\left(\frac{1}{n}\right)-\lim_{n\to\infty}f(2n)$$

$$\lim_{n\to\infty}\left(\tan^{-1}\left(\frac{1}{n^2}+\frac{1}{n}+1\right)-\frac{\pi}{4}\right)-\lim_{n\to\infty}\left(\tan^{-1}(4n^2+2n+1)-\frac{\pi}{4}\right)$$

$$=\left(\frac{\pi}{4}-\frac{\pi}{4}\right)-\left(\frac{\pi}{2}-\frac{\pi}{4}\right)=-\frac{\pi}{4}$$

$\because \quad f(n)=\tan^{-1}(n^2+n+1)-\frac{\pi}{4}$

$$\lim_{n\to\infty}\left[\tan^{-1}(n^2+n+1)-\frac{\pi}{4}\right]$$

$$\lim_{n\to\infty}\left[\frac{\pi}{2}-\frac{\pi}{4}\right]=\lim_{n\to\infty}\left[\frac{\pi}{4}\right]=0$$

$$\tan\left(f(n)+\frac{\pi}{4}\right)=n^2+n+1$$

$$\cot\left(f(n)-\frac{\pi}{4}\right)=\cot\left(\tan^{-1}(n^2+n+1)-\frac{\pi}{2}\right)$$

$$=-\tan\tan^{-1}(n^2+n+1)=-(n^2+n+1)$$

2.(ACD)

$S_1: x^2+y^2+4y-1=0$ centre $C_1(0,-2)$; $r_1=\sqrt{5}$

$S_2: x^2+y^2+6x+y+8=0$ centre $C_2\left(-3,-\frac{1}{2}\right)$; $r_2=\frac{\sqrt{5}}{2}$

$S_3: x^2+y^2-4x-4y-37=0$ centre $C_3(2,2)$; $r_3=3\sqrt{5}$

$C_1C_2=r_1+r_2$ $\quad$ S_1 and S_2 touch externally

$C_2C_3=r_3-r_2$ $\quad$ S_2 and S_3 touch internally

$C_1C_3=r_3-r_1$ $\quad$ S_1 and S_3 touch internally

Point of contact of S_1 and S_2 is $P_1(-2,-1)$

Point of contact of S_2 and S_3 is $P_2(-4,-1)$

Point of contact of S_1 and S_3 is $P_3(-1,-4)$

Common tangent at $P_1 \Rightarrow 2x-y+3=0$

Common tangent at $P_2 \Rightarrow 2x+y+9=0$

Common tangent at $P_3 \Rightarrow x+2y+9=0$

Intersection of common tangents of circles (pairwise) is (–3, –3)

3.(AD) For given system of equations $\Delta = 0$

$$\Delta_1 = a + 7b - 13c$$

$$\Delta_2 = a + 7b - 13c$$

$$\Delta_3 = 0$$

For atleast one solution ;

$$a + 7b - 13c = 0 \qquad \ldots\ldots(i)$$

Option A: $\Delta = 0$; $x + 2y + z = a$

$x + 2y + z = b/2$

$x + 2y + z = c/3$

For atleast one solution

$$a = \frac{b}{2} = \frac{c}{3}$$

But this relation is not satisfying equation (i)

Option B: Consistent when

$a + b + 3c = 0$

And there are non-zero values of

(a, b, c) that simultaneously satisfy

$a + b + 3c = 0$

And : $a + 7b = 13c$

Option C: $\Delta \neq 0$ consistent system

Option D: $\Delta = 0$; $a = -\frac{b}{2} = 2c$

But not satisfying equation (i)

$\therefore$ System is inconsistent

4.(ABC)

Let $A(at_1{}^2, 2at_1)$; $B(at_2{}^2, 2at_2)$

$$\therefore \quad -t_1 - \frac{2}{t_1} = -t_2 - \frac{2}{t_2}$$

$$t_1 t_2 = 2$$

$$M = \left(\frac{a(t_1{}^2 + t_2{}^2)}{2}, a(t_1 + t_2)\right)$$

Let $C(h, k)$

$$h = 2a + a(t_1^2 + t_1 t_2 + t_2^2) = 4a + a(t_1^2 + t_2^2)$$

$$k = -at_1 t_2 (t_1 + t_2) = -2a(t_1 + t_2)$$

Mid-point of $MC = (\alpha, \beta) = \left(\frac{\frac{a(t_1^2 + t_2^2)}{2} + h}{2}, \frac{a(t_1 + t_2) + k}{2}\right)$

$$2\alpha = \frac{a(t_1^2 + t_2^2)}{2} + h,\ 2\beta = a(t_1 + t_2) + k\ = a(t_1 + t_2) - 2a(t_1 + t_2)$$

$$2\alpha = \frac{a(t_1^2 + t_2^2)}{2} + 4a + a(t_1^2 + t_2^2)$$

$$= \frac{3}{2}a(t_1^2 + t_2^2) + 4a$$

$$= \frac{3}{2}a[(t_1 + t_2)^2 - 2t_1t_2] + 4a$$

$$= \frac{3}{2}a\left[\frac{4\beta^2}{a^2} - 4\right] + 4a$$

$$2\alpha = \frac{6\beta^2}{a} - 6a + 4a$$

$$2\alpha = \frac{6\beta^2}{a} - 2a$$

$$\alpha = \frac{3\beta^2}{a} - a$$

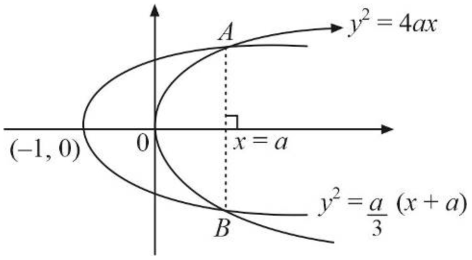
Locus of (α, β) is

$$x = \frac{3y^2}{a} - a$$

$$\frac{a}{3}(x + a) = y^2$$

$$2\beta = -a(t_1 + t_2)$$

$$t_1 + t_2 = -\frac{2\beta}{a}$$



$$\frac{a}{3}(x + a) = 4ax \quad \Rightarrow \quad x + a = 12$$

$$x = \frac{a}{11}$$

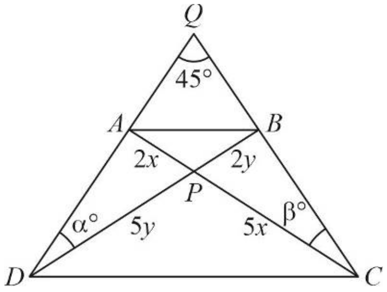
5.(ABC)

ΔABP and ΔCDP are similar,

let $AP = 2x$ and $BP = 2y$

then $CP = 5x$ and $DP = 5y$

Area of trapezium $ABCD = \frac{49}{2}xy$



$\tan\alpha = \frac{2x}{5y}, \ \tan\beta = \frac{2y}{5x}$

$$\alpha + \beta = 45°$$

$$\Rightarrow \quad \frac{10(x^2 + y^2)}{21xy} = 1$$

$$\Rightarrow \quad 10(x^2 + y^2) = 21xy$$

Also, $AB^2 = AP^2 + BP^2$

$$x^2 + y^2 = 4 \qquad \therefore \qquad xy = \frac{40}{21}$$

Hence, Area of $ABCD$ $= \frac{49}{2} \times \frac{40}{21} = \frac{140}{3}$

Area of triangle PCB $= \frac{1}{2} \cdot 2y\, 5x = 5xy = \frac{200}{21}$

Difference of CP and PD is $= 5|x - y| = \frac{10}{\sqrt{21}}$

6.(ABC)

By *LH* rule; $\lim\limits_{t\to x}\dfrac{e^t f(x)-e^x f'(t)}{(f(x))^2}=3$

$\dfrac{e^x f(x)-e^x f'(x)}{f^2(x)}=3\,;\quad d\left(\dfrac{e^x}{f(x)}\right)=3$

Integrate

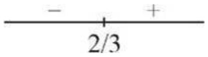
$\dfrac{e^x}{f(x)}=3x+c \quad \because \quad f(0)=1\,; \quad \dfrac{1}{1}=c\Rightarrow c=1$

$\therefore \quad \dfrac{e^x}{f(x)}=3x+1 \qquad \therefore \quad f(x)=\dfrac{e^x}{3x+1}$

$f'(x)=\dfrac{(3x+1)e^x-3e^x}{(3x+1)^2}$

$f'(x)=\dfrac{e^x(3x-2)}{(3x+1)^2}$

$f'(0)=-2$



In, $x\in(1,2)\,f(x)$ increasing function

$f(x)>f(1)$

$\dfrac{e^x}{3x+1}>\dfrac{e}{4}\,; \qquad e^{x-1}>\dfrac{3x+1}{4}$

7.(4) $\displaystyle\int_0^{\pi/4}\frac{1}{\sin^{3/4}x\cdot\cos^{5/4}x}\,dx$

$$\int_0^{\pi/4}\frac{1}{\tan^{3/4}x\cdot\cos^2 x}\,dx=\int_0^{\pi/4}\frac{\sec^2 x\,dx}{\tan^{3/4}x}$$

Substitute $\tan x=t$

$\sec^2 x\,dx=dt$

$$=\int_0^1\frac{1}{t^{3/4}}\,dt=\int_0^1 t^{-3/4}dt=4(t^{1/4})_0^1=4$$

8.(1) Differentiate $F(x)$

$$F'(x)=\begin{vmatrix}\sin\alpha & \cos(x+\alpha) & \sin(x+\alpha)\\ \sin\beta & \cos(x+\beta) & \sin(x+\beta)\\ \sin\gamma & \cos(x+\gamma) & \sin(x+\gamma)\end{vmatrix}+0+0$$

$C_2\to C_2+\sin x\,C_1$

$C_3\to C_3-\cos x\,C_1$

$F'(x)=0$

$\therefore \quad F(x)$ is independent of x

$\therefore \quad F'(x)=0$

9.(9) $a = {}^5P_3 = 5 \times 4 \times 3 = 60$

$b = 150$

$$\frac{a \cdot b}{1000} = \frac{60 \times 150}{1000} = \frac{9000}{1000} = 9$$

10.(1) $\frac{dx}{dy} = \frac{1}{(e^{\sin x} - y)\cot x}$; $\qquad \frac{dy}{dx} = (e^{\sin x} - y)\cot x$

$$\frac{dy}{dx} + y\cot x = e^{\sin x} \cdot \cot x$$

On multiplying by sin$x$

$$\sin x \frac{dy}{dx} + y\cos x = e^{\sin x} \cdot \cos x\ ; \quad \frac{d(y\sin x)}{dx} = e^{\sin x} \cdot \cos x$$

$$\int d(y\sin x) = \int e^{\sin x} \cdot \cos x\, dx\ ; \quad y\sin x = e^{\sin x} + c$$

$$y\left(\frac{\pi}{2}\right) = e + c = e - 1\ ; \quad c = -1$$

$$y = \frac{e^{\sin x} - 1}{\sin x}\ ; \quad \lim_{x \to 0} y = \lim_{x \to 0} \frac{e^{\sin x} - 1}{\sin x} = 1$$

11.(3) on Putting $x = 0$, $y = 0$ in functional equation we will get $f(0) = 0$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x) + f(h) + xh(x+h) - f(x)}{h} \qquad = \lim_{h \to 0} \frac{f(h)}{h} + \lim_{h \to 0} x(x+h)$$

$$= \lim_{h \to 0} \frac{f(h) - f(0)}{h} + x^2 \quad = f'(0) + x^2$$

$$\therefore \quad f'(x) = -1 + x^2 \qquad \therefore \quad \log_2(f'(3)) = 3$$

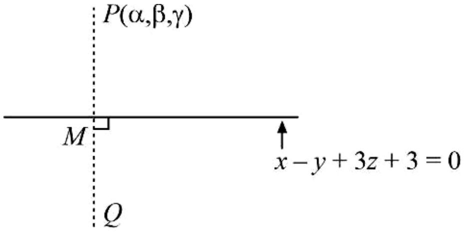
12.(44) M lies on y-axis

So for M point $x = z = 0$

M satisfies the plane $x - y + 3z + 3 = 0$

Hence $y = 3$

M will be (0, 3, 0)



Let the point P is (α, β, γ)

Then Q will be $(-\alpha, -\beta + 6, -\gamma)$

It satisfies $\quad x + y + z = 0$

$\alpha + \beta + \gamma = 6$

Also, DR's of PM is $(\alpha - 0,\ \beta - 3,\ \gamma - 0) \equiv (1, -1, 3)$

Hence, $\frac{\alpha}{1} = \frac{\beta - 3}{-1} = \frac{\gamma}{3} = k$; $\qquad \alpha = k,\ \beta = 3 - k,\ \gamma = 3k$

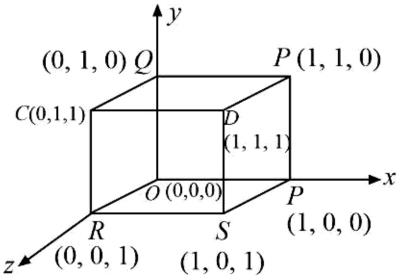
$\alpha + \beta + \gamma = 3k + 3 = 6$; $\quad k = 1$

$\alpha = 1,\ \beta = 2, \gamma = 3$

$P(1,2,3),\ Q(-1,4,-3)$; $\quad (PQ)^2 = 44$

13.(1) Let $\hat{l} = l\,\hat{i} + m\,\hat{j} + n\hat{k}$ where $l^2 + m^2 + n^2 = 1$

Unit vector along 'd_1' diagonal is $\hat{d}_1 = \dfrac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$



Similarly $\hat{d}_2 = \dfrac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

$$\hat{d}_3 = \frac{-\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$$

$$\hat{d}_4 = \frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$$

$$\therefore \quad [(\hat{l}\cdot\hat{d}_1)^2 + (\hat{l}\cdot\hat{d}_2)^2 + (\hat{l}\cdot\hat{d}_3)^2 + (\hat{l}\cdot\hat{d}_4)^2] \quad = \left[\frac{4}{3}(l^2+m^2+n^2)\right] = \left[\frac{4}{3}\right] = 1$$

14.(22) $X = \sum_{r=1}^{10} r^2 \cdot ({}^{10}C_r)^2$

$$\sum_{r=1}^{10} r^2 \cdot \left(\frac{10}{r}\cdot {}^{9}C_{r-1}\right)^2 = 100 \cdot \sum_{r=1}^{10} ({}^{9}C_{r-1})^2$$

$$X = 100 \;\cdot\; {}^{18}C_9$$

$$\frac{X}{221000} = \frac{100\cdot 11\cdot 13\cdot 17\cdot 20}{13\cdot 17\cdot 1000} = 22$$

15.(B) $\dfrac{x^2}{x^2-1} \ge 0$

$x^2 - 1 > 0$ or $x^2 = 0$

$+ \quad - \quad - \quad +$

$-1 \quad 0 \quad 1$

$E_1 : x \in (-\infty,-1) \cup \{0\} \cup (1,\infty)$

Also, $\dfrac{x^2}{x^2-1} = 1 + \dfrac{1}{x^2-1}$

So, $\dfrac{x^2}{x^2-1} > 1$ or $\dfrac{x^2}{x^2-1} = 0$

$$\tan^{-1}\left(\frac{x^2}{x^2-1}\right) > \frac{\pi}{4} \text{ or } \tan^{-1}\left(\frac{x^2}{x^2-1}\right) = 0$$

Domain of $f(x)$ is $E_1 : x \in (-\infty,-1) \cup \{0\} \cup (1,\infty)$

Range of $f(x)$ is $\{0\} \cup (\pi/4, \pi/2)$

For E_2; $\log_e \tan^{-1}\left(\dfrac{x^2}{x^2-1}\right)$ is real

$$\tan^{-1}\left(\frac{x^2}{x^2-1}\right) > 0; \qquad \frac{x^2}{x^2-1} > 0$$

$+ \quad - \quad - \quad +$

$-1 \quad 0 \quad 1$

$x \in (-\infty,-1) \cup (1,\infty)$

Domain of $g(x)$ is $x \in (-\infty,-1) \cup (1,\infty)$

Range of $g(x)$ is $(\log_e \pi/4,\ \log_e \pi/2)$

16.(A) There are 8 Boys and 6 Girls

$N_1 = {}^8C_3 \times {}^6C_3 = 1120$; $N_2 = {}^8C_2 \times {}^6C_3 = 560$

$N_3 = {}^8C_7 \times {}^6C_3 + {}^8C_8 + {}^6C_2 = 175$

(iv) If G_2 and B_3 can't be selected that means we have to select girls from 5 girls and to select boys from 7 boys

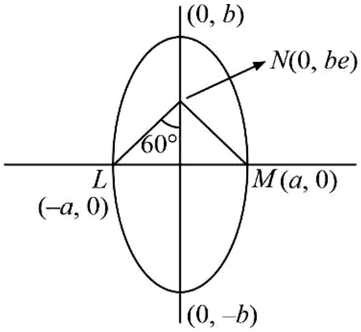
Possible ways $= (3 \text{ Boys}, 2 \text{ Girls}) + (4 \text{ Boys}, 1 \text{ Girl}) + (5 \text{ Boys})$

$= {}^7C_3 \times {}^5C_2 + {}^7C_4 \times {}^5C_1 + {}^7C_5 = 350 + 175 + 21 = 546$

17.(C) $E : \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ $a < b$

$\dfrac{a}{be} = \tan 60° = \sqrt{3}$

$\dfrac{a}{b} = \sqrt{3}e$ ……(i)



$\dfrac{a^2}{b^2} = 1 - e^2 = 3e^2$

$4e^2 = 1 \quad \Rightarrow \quad e = \dfrac{1}{2}$

Area of $\Delta LMN = \dfrac{1}{2} \times 2a \times be = \dfrac{ab}{2} = \sqrt{3}$

$\Rightarrow \quad ab = 2\sqrt{3}$

$\dfrac{a}{b} = \sqrt{3}e = \dfrac{\sqrt{3}}{2} \quad \Rightarrow \quad a = \dfrac{b\sqrt{3}}{2}$

$ab = \dfrac{b^2\sqrt{3}}{2} = 2\sqrt{3}$; $b = 2$; $a = \sqrt{3}$

(2) Length of minor axis = 2a $= 2\sqrt{3}$

(3) Distance between foci is $2be = 2$

(4) The length of latus rectum $= \dfrac{2a^2}{b} = \dfrac{2 \times 3}{2} = 3$

18.(B) $f_1(x) = 2\tan^{-1} x$ when $|x| \le 1$ It is continuous and differentiable at $x = 0$

$f_2(x) = 2|x| - 1$ It is continuous but not differentiable at $x = 0$

$$f_3(x) = \begin{cases} \dfrac{1}{2x+1} & , \ x > 0 \\ \dfrac{1}{2x-1} & , \ x < 0 \\ 1 & , \ x = 0 \end{cases}$$, It is continuous at $x = 0$

$$f_4(x) = \begin{cases} 0 & , \ x > 0 \\ 0 & , \ x < 0, \\ 0 & , \ x = 0 \end{cases}$$ It is continuous and differentiable everywhere